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How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

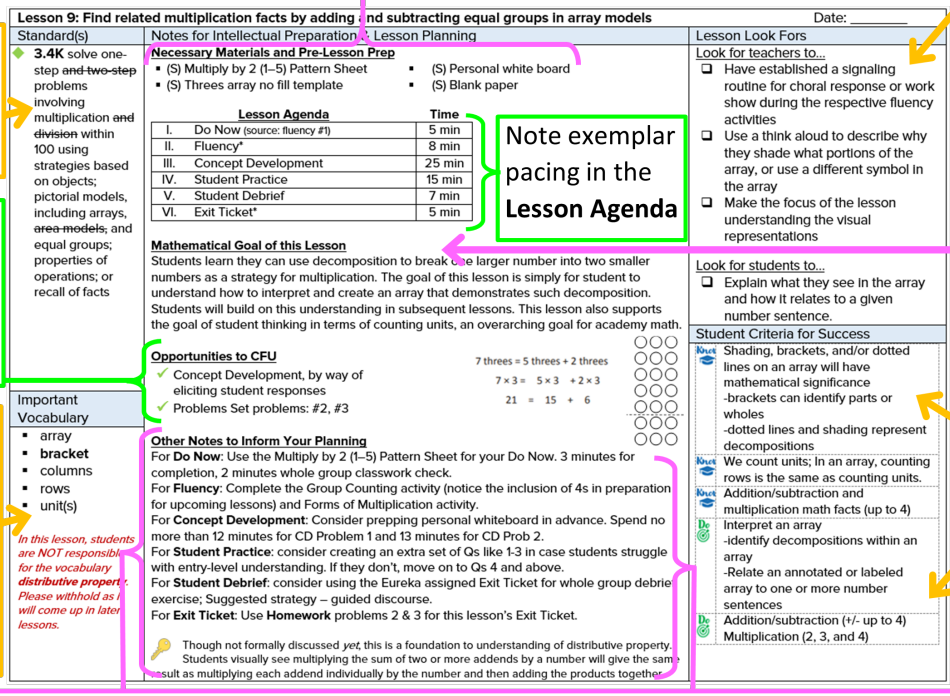
Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**



Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models Date: _____

Standard(s)

3.4K solve one-step ~~and two-step~~ problems involving multiplication ~~and division~~ within 100 using strategies based on objects; pictorial models, including arrays, ~~area models,~~ and equal groups; properties of operations; or recall of facts

Important Vocabulary

- array
- **bracket**
- columns
- rows
- unit(s)

*In this lesson, students are NOT responsible for the vocabulary **distributive property**. Please withhold as it will come up in later lessons.*

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- (S) Multiply by 2 (1–5) Pattern Sheet
- (S) Threes array no fill template
- (S) Personal white board
- (S) Blank paper

Lesson Agenda		Time
I.	Do Now (source: fluency #1)	5 min
II.	Fluency*	8 min
III.	Concept Development	25 min
IV.	Student Practice	15 min
V.	Student Debrief	7 min
VI.	Exit Ticket*	5 min

Mathematical Goal of this Lesson

Students learn they can use decomposition to break the larger number into two smaller numbers as a strategy for multiplication. The goal of this lesson is simply for student to understand how to interpret and create an array that demonstrates such decomposition. Students will build on this understanding in subsequent lessons. This lesson also supports the goal of student thinking in terms of counting units, an overarching goal for academy math.

Opportunities to CFU

- ✓ Concept Development, by way of eliciting student responses
- ✓ Problems Set problems: #2, #3

7 threes = 5 threes + 2 threes
7 × 3 = 5 × 3 + 2 × 3
21 = 15 + 6

Other Notes to Inform Your Planning

For **Do Now**: Use the Multiply by 2 (1–5) Pattern Sheet for your Do Now. 3 minutes for completion, 2 minutes whole group classwork check.
For **Fluency**: Complete the Group Counting activity (notice the inclusion of 4s in preparation for upcoming lessons) and Forms of Multiplication activity.
For **Concept Development**: Consider prepping personal whiteboard in advance. Spend no more than 12 minutes for CD Problem 1 and 13 minutes for CD Prob 2.
For **Student Practice**: consider creating an extra set of Qs like 1-3 in case students struggle with entry-level understanding. If they don't, move on to Qs 4 and above.
For **Student Debrief**: consider using the Eureka assigned Exit Ticket for whole group debrief exercise; Suggested strategy – guided discourse.
For **Exit Ticket**: Use **Homework** problems 2 & 3 for this lesson's Exit Ticket.

Though not formally discussed *yet*, this is a foundation to understanding of distributive property. Students visually see multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

Lesson Look Fors

Look for teachers to...

- ❑ Have established a signaling routine for choral response or work show during the respective fluency activities
- ❑ Use a think aloud to describe why they shade what portions of the array, or use a different symbol in the array
- ❑ Make the focus of the lesson understanding the visual representations

Look for students to...

- ❑ Explain what they see in the array and how it relates to a given number sentence.

Student Criteria for Success

Know: Shading, brackets, and/or dotted lines on an array will have mathematical significance
-brackets can identify parts or wholes
-dotted lines and shading represent decompositions

Know: We count units; In an array, counting rows is the same as counting units.

Know: Addition/subtraction and multiplication math facts (up to 4)

Do: Interpret an array
-identify decompositions within an array
-Relate an annotated or labeled array to one or more number sentences

Do: Addition/subtraction (+/- up to 4)
Multiplication (2, 3, and 4)

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

UNIT SYNOPSIS

In this unit, we will explore **exponential functions** and **logarithmic functions** and use them to model real-life situations. Exponential functions are in the form of $y = ab^x$ where the independent variable is in the exponent. The inverse function of exponential functions are logarithmic functions. Logarithmic functions are in the form of $y = a \log_b x$ where $b > 0$ and $b \neq 1$ for all $x > 0$. This unit will cover basic properties of exponential and logarithmic functions, their transformations, how to solve such equations, and apply the listed knowledge and skills to model growth and decay of natural phenomena, such as population of a region, radioactivity in an area, finances, and much more.

This unit features two overarching themes, exponential and logarithmic functions.

CONTENT STANDARDS

Below are the standards addressed in this unit.

Texas Essential Knowledge and Skills (TEKS)	
Knowledge and Skills	**Student Expectations (SE)**
(2) Functions The student uses process standards in mathematics to **explore**, **describe**, and **analyze** the **attributes of functions**. The student makes connections between **multiple representations** of functions and **algebraically constructs** new functions. The student analyzes and uses functions to **model** real-world problems.	**(2.F)** Graph exponential, logarithmic, ~~rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined~~ functions, ~~including step functions~~. **(2.G)** Graph functions, including exponential, logarithmic, ~~sine, cosine, rational, polynomial, and power functions~~ and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. **(2.I)** Determine and analyze the key features of exponential, logarithmic, ~~rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions~~, ~~including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing. **(2.J)** Analyze and describe end behavior of functions, including exponential, logarithmic, ~~rational, polynomial, and power functions,~~ using infinity notation to communicate this characteristic in mathematical and real-world problems. **(2.N)** Analyze situations modeled by functions, including exponential, logarithmic, ~~rational, polynomial, and power functions~~, to solve real-world problems.
(5) Algebraic Reasoning The student uses process standards in mathematics to **evaluate expressions**, **describe patterns**, **formulate models**, and **solve equations and inequalities** using properties, procedures, or algorithms.	**(5.G)** Use the properties of logarithms to evaluate or transform logarithmic expressions. **(5.H)** Generate and solve logarithmic equations in mathematical and real-world problems. **(5.I)** Generate and solve exponential equations in mathematical and real-world problems.

[1]Parts of standard that are crossed out are not taught in this unit but will be taught in future units.

Focus on Disciplinary Literacy 	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → Math Supports	L1	L2	L3	L4	L5	L6	L7	L8
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓		✓	✓	✓	✓	✓	
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections		✓				✓	✓	✓
makes use of graphic organizers	Graphic Organizers	✓	✓	✓	✓			✓	✓
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives		✓	✓		✓		✓	
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓	✓	✓	✓	✓	✓	✓
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓	✓	✓	✓
includes strategies that support language development									
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓					
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share				✓			✓	✓
includes sentence stems to support students with explanations	- Sentence Stems								✓
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics								
includes websites or equipment that enhances the lesson	Technological Support	✓	✓	✓	✓	✓	✓	✓	✓
content can be presented in different forms									
uses hands-on tools or manipulatives to represent the math	- Concrete								
uses drawings to represent the math	- Pictorial	✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓

ROADMAP

AT A GLANCE: Unit 3 - Exponential and Logarithmic Functions			
Day	Date	Lesson	Lesson Title
1		1	Exponential Functions
2		2	Exponential Modeling
3			*Unit 3 Success Day Alpha – Review Exponential Functions*
4		3	Logarithmic Functions
5		4	Evaluating Logarithmic Functions
6		5	Combining and Expanding Logarithmic Expressions
7			*Unit 3 Success Day Beta – Review Logarithmic Functions*
8		6	Solving Exponential Equations
9		7	Solving Logarithmic Equations
10		8	Application of Exponential and Logarithmic Functions (Review)
11			*Unit 3 Success Day Gamma – Review topics based on your data*
12			*Unit 3 Success Day Delta – Unit Assessment Review*
13			End of Unit 3 Assessment

Date: _____

Lesson 1: Exponential Functions

Standard(s)

- **(2.F)** Graph exponential, logarithmic, ~~rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined~~ functions~~, including step functions~~.
- **(2.G)** Graph functions, including exponential~~, logarithmic, sine, cosine, rational, polynomial, and power functions~~ and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems.

Important Vocabulary

- **Exponential Function**
- **Exponential Growth**
- **Exponential Decay**
- Domain
- Range
- Horizontal Asymptote
- End Behavior
- Limit Notation

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos (optional)
- Document camera
- Colored pencils
- Student laptops (optional)

Lesson Structure:

- Do Now (7 min)
- INM (33 min)
- Student Practice (12 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will engage in a Desmos exploration of parts of the exponential function, $f(x) = a \cdot b^x$, by making hypotheses about how each variable adjusts the function. Then, explore transformations of exponential functions.

Opportunities to CFU

- ✓ How does a and b affect the scale of an exponential function? (INM)
- ✓ What does b need to be for an exp. function to grow? Decay? (INM)
- ✓ How do you determine horizontal asymptotes of an exponential function?
- ✓ How do parameters a, b, c, and d affect the scale and location of an exponential function? (INM)

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam.

10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e-2$ (B) $2e$ (C) $\frac{e}{2}-1$ (D) $\frac{e-1}{2}$ (E) $e-1$

Lesson Look Fors

Look for teachers to…

- ❑ Facilitate exploration of exponential functions via Desmos.
- ❑ Ask probing questions on how parameters a, b, c, and d affect the parent exponential function (mid to end of INM).
- ❑ Ask probing questions on local and end behavior of an exponential function.

Look for students to…

- ❑ Explain how a, b, c, and d affect the scale and location of an exponential parent function.
- ❑ Explain how the values of $b \neq 0$ *and* 1, determine exponential growth or decay.
- ❑ Describe local and end behavior.
- ❑ **Optional**: students can do the exploration on their own, if possible.

Student Know/Do Chart

Know	Values of a, b, c, and d affect shape and scale of an exponential function.
Know	Exponential functions have horizontal asymptotes.
Do	State the transformations of an exponential function.
Do	Describe key functional characteristics of exponential functions.

Date: _____

Lesson 2: Exponential Modeling

Standard(s)

- **(2.N)** Analyze situations modeled by functions, including exponential, logarithmic, ~~rational, polynomial, and power functions~~, to solve real-world problems.
- **(5.I)** Generate and solve exponential equations in mathematical and real-world problems.

Important Vocabulary

- **Exponential Modeling**
- **Compound Interest**
- **Annually**
- **Semi-Annually**
- **Quarterly**
- **Monthly Daily**
- **Continuously**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos (optional)
- Document camera
- Student laptops (optional)

Lesson Structure:

- Do Now (6 min)
- INM (32 min)
- Student Practice (10 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

This lesson is building onto yesterday's introduction to exponential functions using the formula $y = a \cdot b^x$. Students will learn how and when to use another exponential function, the simple interest formula: $A = P(1+r)^t$ leading to the continuous compound interest formula: $P = P_o e^{r \cdot t}$.

Opportunities to CFU

- ✓ How do you represent a rate of growth/decay in an exponential model? (start of INM)
- ✓ How does simple interest become compound interest? (mid-INM)
- ✓ What type of compound interest yields more dividends in the long run (end of INM)?

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to model exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4) to describe and make inferences in contextual settings.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam.

Focus on Disciplinary Literacy

INM

81. Water is pumped into a tank at a rate of $r(t) = 30\left(1 - e^{-0.16t}\right)$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

(A) 380 gallons
(B) 420 gallons
(C) 829 gallons
(D) 1220 gallons
(E) 1376 gallons

Lesson Look Fors

Look for teachers to…

- ❑ Facilitate the connections between the exponential parent function to simple interest in the **Expanding the Exponential Function** and **Helpful Hints with Word Problems** charts.
- ❑ Facilitates how simple interest becomes compound interest.

Look for students to…

- ❑ Completing the charts and making connections from the exponential function to simple interest.

Student Know/Do Chart

Know — Exponential functions can be used to model exponential growth/decay as well as finances involving compound interest.

Know — Certain percentages or rates of change allow models to grow or decay over an indefinite period of time.

Do — Write models of exponential growth or decay given in the context of a problem or situation.

Do — Compare and use exponential models to answer problems involving finances.

Date: _____

Lesson 3: Logarithmic Functions

Standard(s)

- **(2.G)** Graph functions, including exponential, logarithmic and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems.
- **(2.I)** Determine and analyze the key features of exponential, logarithmic such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.

Important Vocabulary

- **Logarithmic Function**
- **Continuity**
- **Increasing**
- **Decreasing**
- Symmetry
- Boundedness
- Absolute/Local Extrema
- Horizontal/Vertical Asymptote
- End Behavior

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Colored pencils
- Document camera
- Desmos (optional)

Lesson Structure:

- Do Now (5 min)
- INM (30 min)
- Student Practice (10 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, the relationship between the exponential function and its inverse, the logarithmic function as well as its key characteristics and transforming logarithmic functions.

Opportunities to CFU

- ✓ What is one property of inverse functions that you recall? (Beginning of INM)
- ✓ What is the relationship between a log and exp function? (mid-INM)
- ✓ How do parameters a, b, c, and d affect the scale and location of an exponential function? (End of INM)

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trig functions (Unit 4).
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam.

Focus on Disciplinary Literacy

INM

4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Lesson Look Fors

Look for teachers to…

- ❑ Engage students in an exploration activity where $y = \log_2 x$ is derived from $y = 2^x$.
- ❑ Facilitate math discourse over how parameters a, b, c, and d affect the scale and location of a log function.

Look for students to…

- ❑ Plot $y = 2^x$ using a table, then applying inverse properties to derive the graph of $y = \log_2 x$.
- ❑ Discuss the effects of parameters a, b, c, and d to a parent logarithmic function.

Student Know/Do Chart

Know	The inverse of an exponential function is a logarithmic function. Hence, their properties should also reflect that relationship.
Know	Transformations of logs are similar as exponentials and other classes of functions.
Do	Given a log function, state its key properties and characteristics.
Do	State and apply transformations of log functions.

Date: _____

Lesson 4: Evaluating Logarithmic Functions

Standard(s)

- **(5.G)** Use the properties of logarithms to evaluate or transform logarithmic expressions.

Important Vocabulary

- **Common Logarithm**
- **Natural Logarithm**
- **Common Logarithmic Rules**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (30 min)
- Student Practice (10 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

This lesson will be entirely about evaluating and manipulating logarithmic equations. The goal for this lesson is to convert between exponential and logarithmic equations and evaluate logarithmic expressions, including the common and natural log.

Opportunities to CFU

- ✓ What is the relationship between logs and exponentials? (start of INM)
- ✓ How can you express a log as an exponential expression? Exponential as a log? (mid-INM)

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

Student Practice a - g

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the **2008 AP Calculus AB released exam**.

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4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking on converting between log and exponential forms.

Look for students to…

- ❑ Reading out loud problems #1-5 in the INM.
- ❑ Converting between log and exponential forms, including the common and natural log.
- ❑ Evaluate log expressions.

Student Know/Do Chart

- Know: Logarithmic and exponential functions are inverses of each other.
- Know: A log can be converted to an exponential form.
- Do: Convert between log and exponential expressions.
- Do: Evaluate log expressions, including the common and natural log.

Date: _____

Lesson 5: Combining and Expanding Logarithmic Expressions

Standard(s)

- **(5.G)** Use the properties of logarithms to evaluate or transform logarithmic expressions.

Important Vocabulary

- **Product Rule**
- **Quotient Rule**
- **Power Rule**
- **Change-of-Base**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (6 min)
- INM (28 min)
- Student Practice (13 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to apply the product, quotient, and power rule for logarithms as well as the change of base rule to rewrite logarithmic expressions.

Opportunities to CFU

- ✓ What do the exponents do when simplifying y^4/y^{11}? (Do Now)
- ✓ What is the order in applying the properties of logarithms when expanding or condensing? (End of INM)

Other Notes to Inform Your Planning

- Applying properties of logarithms is important when solving exponential or logarithmic equations. In calculus, students will need to apply rules of log prior to differentiating or integration functions.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam.

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22. Let f be the function defined by $f(x) = \frac{\ln x}{x}$. What is the absolute maximum value of f?

(A) 1

(B) $\frac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

Lesson Look Fors

Look for teachers to…

- ❑ Leverage the Do Now activity as it connects directly with the topic for this lesson.
- ❑ Explore with students the informal proofs of the logarithmic properties.

Look for students to…

- ❑ Complete the Do Now as much as they can and name at least one exponent rule from the Do No activity.
- ❑ Working out the proofs for each property individually or in groups.

Student Know/Do Chart

Know	Just like properties of exponents, logarithms have similar properties.
Know	Properties of logarithms must be applied in a specific order.
Do	Apply the conjugate theorem. Use properties of logs to expand a single logarithmic expression.
Do	Use properties of logs to condense log expressions.

Date: _____

Lesson 6: Solving Exponential Equations

Standard(s)

- **(5.I)** Generate and solve exponential equations in mathematical and real-world problems.
- **(2.N)** Analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems.

Important Vocabulary

- **Exponential Equation**
- One-to-One
- Extraneous Solution
- No Solution

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (6 min)
- INM (26 min)
- Student Practice (16 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

We started this unit by examining both exponential and logarithmic functions graphically, how to model them, how to manipulate them, and now students will focus on how to solve equations involving exponential and logarithmic functions.

Opportunities to CFU

- ✓ What if the bases of the exponents aren't the same? (start of INM)
- ✓ How do we isolate the independent variable when it is an exponent? (mid-INM)
- ✓ When are solutions involving a log erroneous? (end of INM)

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

Student Practice Part 2

- When solving equations, we need to apply the reverse of the order of operations. In Unit 4, students will apply this same technique when solving trigonometric equations.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam. This FRQ problem requires students to solve the particular solution involving logarithms after integration.

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

Lesson Look Fors

Look for teachers to…

- ❑ Gradually releasing students to solving exp equations after explaining the 1-1 property.
- ❑ Activates prior knowledge from lesson 5 when solving t (2 examples from INM).
- ❑ Guides students to rewrite some exponential expressions by substitution.

Look for students to…

- ❑ Applying the 1-1 property to solve exp equation in the INM.
- ❑ Applies logarithmic properties to solve exponential equations.
- ❑ Explain how some solutions to an exponential equation can be erroneous.

Student Know/Do Chart

Know	The 1-1 property allows us to quickly find solutions to equations of the form $f(x) = f(y)$.
Know	The reverse order of operations is used when solving for the independent variable.
Do	Solve exponential equations using the 1-1 property, even when bases are not the same.
Do	Apply properties of log to solve exponential equations.
Do	Determine which solutions involving logarithms are erroneous.

Date: _____

Lesson 7: Solving Logarithmic Equations

Standard(s)

- **(5.H)** Generate and solve logarithmic equations in mathematical and real-world problems.
- **(2.N)** Analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems.

Important Vocabulary

- **Logarithmic Equation**
- One-to-One
- Extraneous Solution
- No Solution

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (6 min)
- INM (26 min)
- Student Practice (16 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

This lesson begins with an example of how logarithms are used in the real world: determining intensity levels of earthquakes. By the end of the lesson, students will be able to solve logarithmic equations; log on one side, and log on both sides.

Opportunities to CFU

- ✓ How can you check if you have an erroneous solution? (INM)
- ✓ When dealing with multiple logs in an equation, what can we apply? (INM)
- ✓ How does the 1-1 property assist in solving logarithmic equations? (INM)

Other Notes to Inform Your Planning

- When solving equations, we need to apply the reverse of the order of operations. In Unit 4, students will apply this same technique when solving trigonometric equations.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the **2012 AP Calculus AB released exam**. This FRQ problem requires students to solve the particular solution involving logarithms after integration.

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

Lesson Look Fors

Look for teachers to…

- ❑ Activate prior knowledge in the Do Now.
- ❑ Connects prior knowledge to solve log equations in the beginning of the INM.

Look for students to…

- ❑ Apply properties of log to solve logarithmic equations.
- ❑ Explain how some solutions to a logarithmic equation can be erroneous.

Student Know/Do Chart

Know — Logarithmic equations can be re-expressed to simpler expressions on one or both sides.

Do — Apply properties of logs to re-write and solve logarithmic equations.

Do — Determine which solutions involving logarithms are erroneous.

Date: _____

Lesson 8: Application of Exponential and Logarithmic Functions (Review)

Standard(s)

- **(2.N)** Analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems.
- **(5.H)** Generate and solve logarithmic equations in mathematical and real-world problems.
- **(5.I)** Generate and solve exponential equations in mathematical and real-world problems.

Important Vocabulary

- Review Vocabulary from the Unit

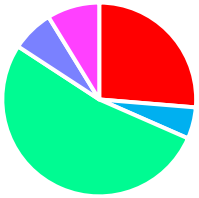
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera
- https://tinyurl.com/GraduatingwithDebt

Lesson Structure:

- Do Now (15 min)
- INM (3 min)
- Student Practice (30 min)
- Debrief (4 min)
- Exit Ticket (5 min)



Mathematical Goal of this Lesson

This lesson is an example of how students may encounter exponential functions in their future lives. The lesson starts with a quick article, which acts as a conversation sparker as well as providing basic information so all students can enter the lesson feeling confident about the topic of student loans.

Opportunities to CFU

- ✓ Write the equation for compound interest on the board and make sure students understand each term. (INM)
- ✓ Student Practice on the table.

Other Notes to Inform Your Planning

This lesson is at the culmination of Unit 3 but it is not designed as a close review of the Unit 3 Exam. The real-world application of this lesson should connect to students' experiences or experiences in their near future which will help them connect to using exponential and logarithmic equations. Hopefully the students will connect with the article, and it will help them apply what they have learned during this unit.

Focus on Disciplinary Literacy

INM Part 2

Lesson Look Fors

Look for teachers to…

- ❑ Utilize the article linked in the lesson and here.
- ❑ Let students do the majority of the conceptual work; only get involved when necessary.

Look for students to…

- ❑ Use information from different scenarios to set up proper exponential equations.
- ❑ Solve exponential equations using a variety of methods.
- ❑ Connect how exponential functions can directly relate to their personal finances.

Student Know/Do Chart

Do: Model real world scenarios with exponential equations.

Do: Use given information to solve for unknown values in financial scenarios with compound interest.

Know: How to apply the compound interest formula to personal finance problems involving college expenses.

Recommended Unit 3 Success Day Material and Resources

Date: _____

To review topics based on your data on Success Days, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer-aligned practice and teacher-led groups.

To review or practice Exponential Functions:

Instructional Resources:

- Math Lab: Graphing Exponential Functions
- Transformations Cheat Sheet
- Exponential Functions and Their Graphs PPT

Content Video Lessons:

- An Introduction to Exponential Functions
- An Introduction to Graphing Exponential Functions
- Transformations of Exponential Functions
- Graphing Exponential Functions: Another Example
- Evaluating and Graphing a Natural Exponential Function

To review or practice Exponential Modeling:

Instructional Resources:

- The Population Problem
- The Meteorology Problem
- The Coffee Cooling Problem
- The Chernobyl Problem
- The Radioactive Decay Problem
- Compound Interest Exploration

Content Video Lessons:

- Half-Life
- Newton's Law of Cooling
- Applying Natural Exponential Functions
- Compound Interest
- Continuously Compounded Interest

To review or practice Logarithmic Functions:

Instructional Resources:

- Logarithmic Functions and Their Graphs PPT

Content Video Lessons:

- Graphing Logarithmic Functions
- Matching Logarithmic Functions with Their Graphs
- Common Logs and Natural Logs
- Domain of a Natural Log Function

To review or practice Evaluating Logarithmic Functions:

Content Video Lessons:

- An Introduction to Logarithmic Functions
- Converting Between Exponential and Logarithmic Functions, Part 1
- Converting Between Exponential and Logarithmic Functions, Part 2
- Converting Between Exponential and Logarithmic Functions, Part 3
- Converting Between Exponential and Logarithmic Functions, Part 4
- Converting Between Exponential and Logarithmic Functions, Part 5
- Evaluating Logarithms, Part 1
- Evaluating Logarithms, Part 2
- Evaluating Logarithms, Part 3
- Using Properties to Evaluate Logarithms

To review or practice Combining and Expanding Logarithmic Expressions:

Instructional Resources:

- Log Properties Investigation
  - o Recording Sheet & Notes
- Properties of Logarithmic Functions PPT

Content Video Lessons:

- Properties of Logarithms
- Expanding Logarithmic Expressions
- Combining Logarithmic Expressions
- Using Change of Base Formulas

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification

Standards

(2A) Determine and analyze the key features of exponential, logarithmic, ~~rational~~, ~~polynomial~~, ~~power~~, ~~trigonometric~~, ~~inverse trigonometric~~, and ~~piecewise defined functions~~, ~~including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.

Specificity

Concepts:

Parent Functions:

- Exponential: $f(x) = a \bullet b^x$
- Logarithmic: $f(x) = \log_b(x)$
- Rational: $f(x) = \frac{1}{x}$
- Polynomial: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$
- Power: $f(x) = a \bullet x^b$
- Trigonometric: $f(x) = \sin(x)$ or $f(x) = \cos(x)$
- Inverse Trigonometric $f(x) = \arcsin(x)$ or $f(x) = \arccos(x)$
- Piecewise-defined functions
- Key features
  - Domain (interval notation*)
  - Range (interval notation*)
  - Symmetry (about x, y, origin)
  - Maximum (local vs. absolute*)
  - Minimum (local vs. absolute*)
  - Extrema*
  - Boundedness*
  - Zeros
  - Asymptotes
  - Intervals of increasing (interval notation*)
  - Intervals of decreasing (interval notation*)

Content:

Including, but not limited to:

- Identify the parent function
- Graph the function
- Determine key features of the function
- Analyze the function by the key features

Notes/Explanations/Examples

Below are the content and skill connections between the Algebra 1 TEKS and the AP standard.

Algebra 2 TEKS

(2A.2.A) graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b x$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum & minimum given an interval;

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Question 5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

(a) $\left.\frac{dW}{dt}\right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$
The tangent line is $y = 1400 + 44t$.
$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411$ tons

2 : $\begin{cases} 1 : \frac{dW}{dt} \text{ at } t = 0 \\ 1 : \text{answer} \end{cases}$

(b) $\frac{d^2W}{dt^2} = \frac{1}{25}\frac{dW}{dt} = \frac{1}{625}(W - 300)$ and $W \geq 1400$
Therefore $\frac{d^2W}{dt^2} > 0$ on the interval $0 \leq t \leq \frac{1}{4}$.
The answer in part (a) is an underestimate.

2 : $\begin{cases} 1 : \frac{d^2W}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$

(c) $\frac{dW}{dt} = \frac{1}{25}(W - 300)$
$\int \frac{1}{W - 300}\, dW = \int \frac{1}{25}\, dt$
$\ln|W - 300| = \frac{1}{25}t + C$
$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$
$W - 300 = 1100e^{\frac{1}{25}t}$
$W(t) = 300 + 1100e^{\frac{1}{25}t}$, $0 \leq t \leq 20$

5 : $\begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } W \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration
Note: $0/5$ if no separation of variables

VERTICAL STANDARDS

This section details the **progression** of key student expectations/standards** in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Algebra 2	Pre-Calculus	AP Calculus AB Essential Knowledge
• **2A.2A** Graph the function $f(x) = b^x$ and $f(x) = \log_b x$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum & minimum given an interval. • **2A.5A** Determine the effects on key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b x$ where b is 2, 10, and e, when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative real values of a, c, and d. • **2A.5B** Formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation. • **2A.5C** Rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations. • **2A.5D** Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations having real solutions. • **2A.5E** Determine the reasonableness of a solution to a logarithmic equation. • **2A.7I** Write the domain and range of a function in interval notation, inequalities, and set notation.	• **(2.F)** Graph exponential, logarithmic, ~~rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined~~ functions, ~~including step functions~~. • **(2.G)** Graph functions, including exponential, logarithmic, ~~sine, cosine, rational, polynomial, and power functions~~ and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. • **(2.I)** Determine and analyze the key features of exponential, logarithmic, ~~rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions~~, ~~including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing. • **(2.J)** Analyze and describe end behavior of functions, including exponential, logarithmic, ~~rational, polynomial, and power functions~~, using infinity notation to communicate this characteristic in mathematical and real-world problems. • **(2.N)** Analyze situations modeled by functions, including exponential, logarithmic, ~~rational, polynomial, and power functions~~, to solve real-world problems. • **(5.G)** Use the properties of logarithms to evaluate or transform logarithmic expressions. • **(5.H)** Generate and solve logarithmic equations in mathematical and real-world problems. • **(5.I)** Generate and solve exponential equations in mathematical and real-world problems.	• **LIM-2.D.1** The concept of a limit can be extended to include infinite limits. • **LIM-2.D.2** Asymptotic and unbounded behavior of functions can be described and explained using limits. • **LIM-2.D.4** Limits at infinity describe end behavior. • **LIM-2.B.2** Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous on all points in their domains. • **FUN-3.A.4** Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions. • **FUN-7.F.1** Specific applications of finding general and particular solutions to differential equations include motion along a line and exponential growth and decay. • **FUN-7.F.2** The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$. The exponential growth and decay model, $\frac{dy}{dt} = ky$, with initial condition $y = y_0$ when $t = 0$, has solutions of the form $y = y_0 e^{kt}$. NOTE: These "essential knowledge" (EK) standards are from The College Board Course Exam Description for AP Calculus AB/BC, not the TEKS.